

# VIBRATION ANALYSIS OF A CRACKED ROTATING SHAFT

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**Abstract:** Shafts are considered as the main component subjected to the hardest stress in efficient rotating equipment such as high speed compressors, pumps and gas turbines, etc. Many errors and defects can appear during the operation that can be considered as serious. Vibration spectrum can be a very good indication for the presence of a fault in the machine including unbalance, misalignment, shaft bending and many more each fault has its own effect on the vibration spectrum. The rapidly fluctuating nature of the results can be considered as a good indication if the presence of a crack since the development of a crack changes the dynamic behavior of the whole system. Cracks can cause the vibration response of the system to change when running, which can give a good indication of the presence for crack detection. This paper shows the vibration analysis of rotating shafts with different conditions; location of the crack, rotational speed of the shaft, crack dimensions, etc. , using matlab and solid works to analyze and simulate the effect of changing the conditions on the spectrum.

**Keywords** – Vibration Analysis, Crack, Shaft, Matlab, Solidworks, Simulation

## 1 INTRODUCTION

Shafts are mechanical components used to transmit motion and torque to a device or machine. They are commonly used to connect components that cannot be connected directly due to distance or to synchronize their motion together. Drive shafts are subjected to stresses as they transmit motion. They must be strong enough to withstand these stress, while avoiding any extra unnecessary weight to avoid increasing the inertia. Shafts are considered among the components subjected to the high stress working conditions in power transmission applications. Shafts are also used in heavy duty rotating equipment such as turbo-shaft engines, steam and gas turbines, compressors, generators, pumps, etc. Although shafts are considered robust and good quality, in operation they are usually susceptible to serious defects that develop without much apparent warning.

Vibrations in any of the rotating machinery is caused by faults such as imbalance, misalignment, cracks, etc. The greatest challenge in the area of condition monitoring is the diagnosis of a fault before it becomes critical. Early detection of faults allows for plant downtime to be planned, and for the necessary personnel and replacement parts to be available. It is therefore important that the system is continuously monitored to assess its condition

even if it was trouble-free when it was started.

## 2 LITERATURE REVIEW

Bloch [1] studied the cracks of shafts takes place due to diverse mechanisms such as high and low cycle fatigue, or stress corrosion. A normal chronology of events leading to total failure due to cracking in a ductile material.. Cracks can be caused by a lot of faults including mechanical stress raisers, such as sharp keyways, abrupt cross-sectional changes, heavy shrink fits, dents and grooves, or metallurgical factors such as flaws in forging, inclusions, porosity and voids.

Fuchs and Stephens [2] emphasized on the studying of fatigue design methods with the intent to attain safe, reliable and economical procedures; with the help of computer aided engineering and digital prototyping.

Ichimonji [3] considered a slant crack usually occurs from fatigue due to torsional moment on a shaft as well as studying the dynamics of a simple rotor for qualitative analysis using a 3D finite element method.

Sekhar [4] Studied the finite element (FEM) analysis of a rotor

bearing system for flexural vibrations considered by including a shaft having a slant crack that has resulted from the fatigue of a shaft due to torsional moment, also developed a flexibility matrix for a crack and the stiffness matrix of a slant cracked element to be used in the FEM analysis of the rotor-bearing system. The frequency spectrum of the steady state response was found to have subharmonic frequency components at an interval frequency corresponding to the torsional frequency

Sekhar and Prabhu [5] discovered that cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as “breathing” cracks. The stiffness of the shaft is most affected when under tension. The “breathing” of the crack results in non-linearity in the vibrational behavior of the rotor. They concentrated in their studies on the effects of acceleration rate, crack depth and position of unbalance on the vibrational behavior of a cracked rotor during run up. Also they can be considered of the opinion that the increase in the 1/2 and 1/3 critical response is a reliable indicator of cracks.

Gasch, R [6] provided a simple but comprehensive survey about the stability behavior of a rotating shaft with a crack, and of the forced vibrations due to imbalance and to the crack.

Subbiah, R., Montgomery [7] stated that cracks that open on the surface are called “surface” cracks, which can be detected easily by techniques such as dye-penetrant, or visual inspection. Cracks that are hidden beneath the surface are called “subsurface” cracks. Special methods such as ultra-sonic, magnetic particle, radiography or shaft voltage drop can be used to detect them. Surface cracks are more dangerous than subsurface cracks on the vibrational behavior of shafts. They concluded that transverse cracks primarily respond to bending, but can also respond to torsion depending on their position, surface cracks have a greater and earlier effect than subsurface cracks.

Bently and Muszynska [8] studied the detection of shaft cracks using signal based methods. In many cases, corrosion, misalignment and heavy side loads have been identified as the causes for shaft cracks in compressors, generators, gears and nuclear coolant pumps. Direct shaft measurements can be considered better and more reliable than bearing housing measurements. Although several researchers have found that 2x component can be a good indicator of shaft cracks.

Allen and Bohanick [9] studied the effect of a crack on pumps as well as practical trial to obtain results from recirculation pumps on a reactor in a plant, to conclude that the bearing housing measurement cannot be relied on for the analysis of a crack.

Werner [10] was also of the opinion that the trend of the 1x component is a better indicator. The reason why the 2x component is present in the spectrum of a cracked shaft is mainly because of the local asymmetric shaft stiffness (due to a crack) and the presence of radial loads. The 2x component is too sensitive to other factors such as side loads, misalignment.

Saavedra and Cuitino [11] recommend observing the shaft 2x component to present a theoretical and experimental analysis to demonstrate that the 2x component of vibration for horizontal shafts at half the first critical speed value can give a good indica-

tion.

Lazzeri et al [12] suggested that monitoring the second harmonic component of the vibration is demonstrated to be a valuable method of early assessing structural degradation. The rationale is shown considering a real case where extensive fracture mechanics.

Bently and Muszynska [13] mentioned that observing the 2x component during start-up/coast-down is more useful than during steady-state operation due to the distinctive changes due to cracks when analyzing.

Sanderson [14] introduced the factors that helped identify a crack in a turbo generator as follows: a large and steadily increasing 1x component due to shaft crack induced unbalance, the temperature gradient within the rotor was quite large from bore to surface, and the crack distorted the distribution of thermal stresses, which resulted in bending of the shaft, a small reduction in generator first and second critical speeds observed during run up, large 2x component at half the generator first critical speed, split first critical speed due to asymmetry in the normally symmetrical rotor (four-pole generator).

Muszynska et al [15] stated that torsional vibrations are excited even by purely radial forces, such as unbalance and misalignment in the case of cracked shafts. Thus, monitoring the torsional vibrations in many horizontal and vertical machines. It can help to detect cracks when 8x, 6x, 4x etc. correspond to the lowest torsional frequency. Researchers feel that with the improvement in transducer and signal conditioning technology, torsional-vibration monitoring of turbo-machines will increase in application.

Dorfman and Trubelja [16] exhaustively cover the subject of torsional vibrations in steam turbines and turbo-generators and its application in crack detection. They stressed on the instrumentation, data acquisition and signal processing aspects of monitoring torsional vibrations.

Ishida et al [17] suggested an alternative indicator for detecting cracks in rotors operating in the transcritical and supercritical ranges. Instead of monitoring 1x or 2x components, they suggested that, in the  $(2\omega_c/3 \text{ to } 2\omega_c)$  range, the 1/2x, 3/2x and 9/2x components should be monitored as they are most sensitive when it comes to cracks (where  $\omega_c$  is the critical speed of the rotor).

Gasch and Liao [18] have patented an orbit-based method of crack detection. Signals that represent flexural vibrations of the shaft are measured as a function of the rotational angle. These signals are transmitted to a signal processor which uses the signals to ascertain the harmonic vibration components with single or double or triple rotational frequencies the processor forms a single, double or triple orbit by means of a vectorial combination of these vibration components.

Plaut et al [19] investigated the transient behavior of a cracked shaft during constant acceleration or deceleration past a critical speed, studied both breathing and gaping cracks. They concluded that the response is minimum when the acceleration or deceleration are rapid through the critical speed zone and the response increases with an increase in crack depth.

Kavarana and Kirk [20] they observed that the maximum transient response is very sensitive to the position of the unbalance the

crack. They constructed a test rig where when unbalance weights are placed at different angles with respect to a crack and the response is measured. Based on studies on a Laval rotor with a small transverse breathing crack,

Bachs Schmid et al [21] presented a robust method for detecting the position and the depth of cracks on rotors. A model-based approach and a least-squares identification method in the frequency domain are used for the locating the crack along the rotor. The crack depth is calculated by comparing the static bending moment due to the rotor weight reaction on the shaft and the alignment conditions in the bearings, to the identified "equivalent" periodic bending moment, which represents a simulation to the crack.

Guo et al [22] have applied the finite element method (FEM) to study the influence of cracks on all three types of shaft vibrations: torsional, axial and lateral. A stiffness matrix ( $12 \times 12$ ) is considered for the crack. They stated that you cannot rely on torsional vibrations by themselves to indicate shaft cracks because of their relatively small magnitudes. Instead, the presence of a strong 1x axial vibrations when applying purely radial excitations can indicate cracks better. Breathing cracks can produce 2x and 3x components in the lateral vibrations.

Mohiuddin and Khulief [23] presented yet another FEM-based crack detection scheme. Introduced a dynamic model for a large scale rotor-bearing system with a cracked shaft, A finite shaft element with a crack was developed using a consistent finite element approach. The model accommodates shafts with tapered portions, multiple disks and bearings. Mathematical models of the rotor and the equations of motion are solved using various techniques.

Park [24] described an advanced method of a crack detection: a new way to localize position and to estimate depth of a crack on rotating shaft. As a first step, the shaft is physically modelled with a finite element method and the dynamic mathematical model is derived using the Hamilton principle; thus, the system is represented by various subsystems. The equations of motion of the shaft with a crack are established by adapting the local stiffness change through breathing and gaping from the crack to an undamaged shaft.

Ostachowicz and Krawczuk [25] presented a mathematical model for the stiffness of a section of shaft containing a gaping transverse crack. They derive a  $5 \times 5$  flexibility matrix. Fracture mechanics is used to derive the stiffness matrix using stress intensity factors due to the crack, and it can be seen that the stiffness matrix has coupled terms. This element can be used in the FE analysis of complex rotors and prediction can be at any location and any speed.

Ratan, S., Baruh [26] defined a vector quantity, called the "residue", for additional details pertaining to the residue. The presence of a crack is indicated by any non-zero value of the quantity at any section. This method was proven to be capable of locating and detecting cracks as small as 4% of the shaft diameter.

Yang et al [27] studied the dynamic characteristics of cracked shaft in the subcritical, trans-critical and supercritical regions. Transient vibrations of the rotor are studied using Holo-spectral method is used to study the transient vibrations of the rotor. Because of the

extreme sensitivity of this technique, the authors claim that it can be used to detect incipient cracks, i.e. even before an actual crack propagates. The experimental results show that due to the fact that the proposed methods provide much more detailed and distinguishable characteristics for diagnosis, they are very efficient for capturing incipient transverse cracks in a rotor. Moreover, these methods are also broad and applicable to all high-speed rotating machines.

Green and Casey [28] presented two theoretical analysis techniques. Using both global and local asymmetry models, they can be used in identifying the most suitable target characteristic for crack detection. The 2x component is proven to be the primary response component. Also, the 2x resonance speed is affected due to a crack usually lowered. As well as using the 2X harmonic response component to indicate a transverse shaft crack.

Meng and Hanh [29] considered time-dependent terms as external excitation forces and analyze, both theoretically and numerically, the approximate dynamic response of a cracked horizontal rotor. The forward and backward whirl for each steady-state harmonic component as well as amplitudes, the shape and orientation of the elliptical orbit and phase of the response signals are analyzed, taking into account the effect of the crack criteria, rotor speed and imbalance.

Chan and Lai [30] discuss the FE-based simulation of a shaft with a transverse crack. They analyze the four possible cases: symmetrical un-cracked shaft; symmetrical cracked shaft; asymmetrical un-cracked shaft; asymmetrical cracked shaft. They stated that the response of the symmetrical cracked is very similar to the response of the asymmetrical un-cracked. At half of the first critical speed they both show resonance. However, the symmetrical cracked shaft also shows resonance at third the critical speed which is not shown in the asymmetrical un-cracked shaft.

Prabhu and Sekhar [31], presented a severity estimation criterion and crack growth monitoring method for cracked shafts in fluid film bearings. The dynamic pressure on the oil film is used as the target criterion. As crack depth and the dynamic pressure are directly proportional. It is concluded that the peak pressure measurements are not the best indicators because of the difficulty involved in their estimations and also because of their insensitivity.

Guang and Gasch [32] investigated the stability of a cracked rotor supported on two axial groove, four lobe and five tilting pad type journal bearings. It was found that, regardless the type of bearing used, the rotor is always unstable in a certain range of speeds. The stability depends on the stiffness ratio for large values of the gravity factor which is used to measure the elasticity of the shaft, while for smaller values it depends on the mass ratio which the ratio of lumped mass at the bearing to that at the center of the shaft. In this range, the shaft stability depends only on the bearing type and operating conditions not on the crack.

Sekhar [33] presented a unique crack detection methodology based on the measurement of the Q factor of a rotor during coast down. The Q factor is the amplification factor of the frequency component is determined from the corresponding plot by the popular "half-power" method. The research emphasized that this parameter, especially for the 2x component. The 2X is the most

sensitive to shaft asymmetry as well as sudden changes of the Q factor during coast-down are good indicators for cracks.

### 3 MATERIALS AND METHODS

The total length of the specimen was 1000mm and 30 mm in diameter.

#### 3.1 Shaft crack shape

As discussed earlier, as shown in Fig (1) a transverse crack caused by material fatigue is a very common defect in rotating equipment that operates for extended periods under heavy load. This type of crack remains the most important type of crack as the machine safety is significantly influenced by its occurrence, Table (1) shows the main parameters of the cracked shaft used in the modeling.

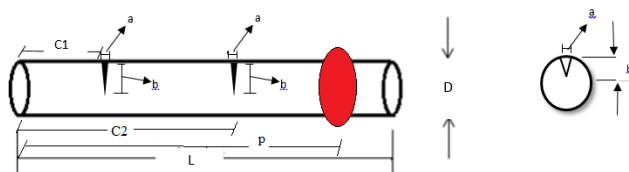


Fig (1) Geometry of a cracked shaft

Table (1) Main parameters of the cracked shaft

Parameters	Value
Mass of shaft (M)	5.57 'kg'
Density ( $\rho$ )	7890 'kg/m <sup>3</sup> '
Length (L)	1000 'mm'
Modulus of Elasticity (E)	210 'Gpa'
Diameter (D)	30 'mm'
Crack position (C <sub>1</sub> )	C <sub>1</sub> 'm' [0.1, 0.2, 0.3, 0.4, 0.5, 0.75 m]
Crack position (C <sub>2</sub> )	C <sub>2</sub> 'm' [0.7, 0.8, 0.9 m]
Crack Width (b)	b 'mm' [0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 1 mm]
Crack depth (a)	a 'mm' [1mm]
Mass position (p)	P 'm' [0.1, 0.3, 0.5, 0.7, 0.9 m]
Rotational speed (N)	N 'Rpm' [50, 100, 250, 500, 750, 1500, 3000, 6000]
Mass (m)	m 'kg' [5, 7, 10, 20, 30 kg]

### 4 RESULTS AND DISCUSSIONS

The Cracked shaft is modeled using the matlab software with the before mentioned parameters which resulted in producing some curves showing the amplitude along the shaft distance, in several modes. We will discuss the representation of the seventh mode in three cases; a case with two bearings supporting both ends of the shaft and one crack along the shaft, a case with three bearings supporting the shaft at both ends and the center of the shaft with two cracks (one between each two bearings), and the last case is with two bearings supporting

the shaft at both ends and two cracks along the shaft varying their position along the shaft.

#### 4.1 TWO BEARINGS WITH TWO CRACKS

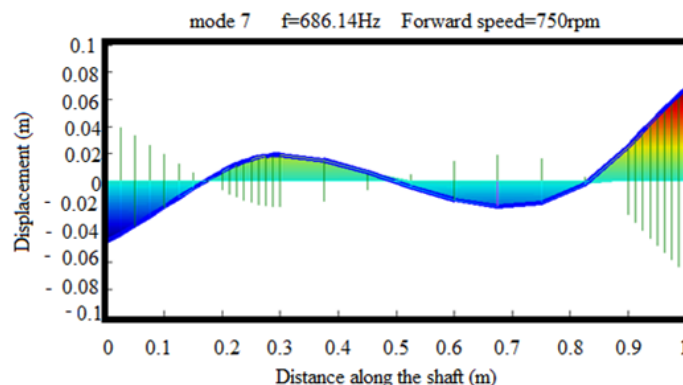


Fig (2) Displacement of shaft – One Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.3m C<sub>2</sub>=0 N=750rpm)

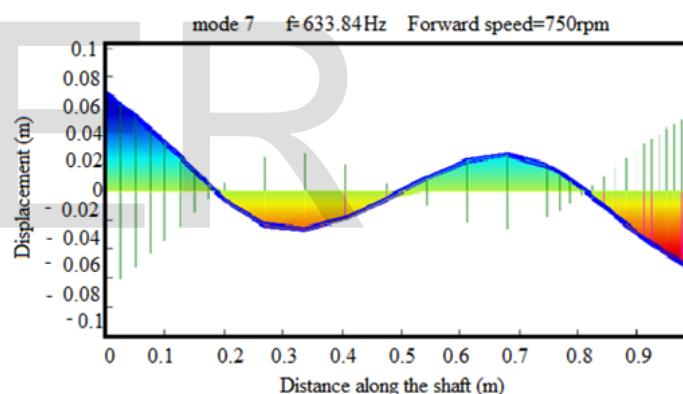


Fig (3) Displacement of shaft – One Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.5m C<sub>2</sub>=0 N=750rpm)

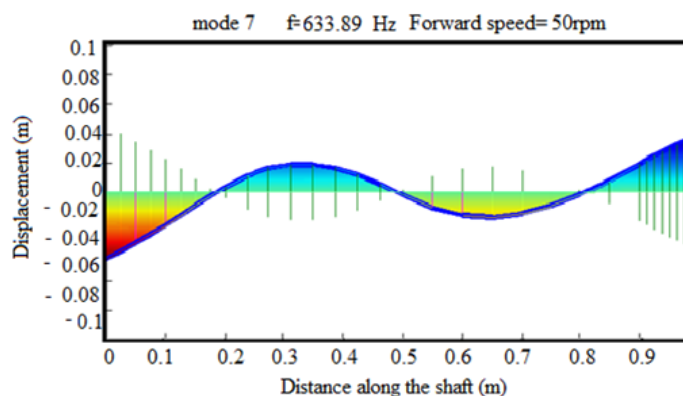


Fig (4) Displacement of shaft – One Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.5m C<sub>2</sub>=0 N=50rpm)

### 4.1.1 EFFECT OF CRACK POSITION VARIATION

The seventh mode of a cracked shaft supported by two bearings at different crack positions (0.3m, 0.4m, 0.5m, and 0.75m) consequently with fixed parameters of the rotational speed of the shaft at (750 rpm), and the crack dimensions constant at (1mm depth and 0.1mm width) shown in (Fig 2). It is shown that the amplitude of the vibration changes according to the distance of the crack along the shaft since the amplitude increases as the crack is located beside the supports.

### 4.1.2 EFFECT OF CRACK WIDTH VARIATION

The seventh mode of a cracked shaft supported by two bearings, at different crack dimensions by changing the width of the crack (0.1mm, 0.2mm, 0.3mm, 0.4mm, 0.5mm, 0.7mm, and 1mm) at the same crack position at the center of the shaft and crack depth at (1mm), and constant shaft rotation speed at (750 rpm) shown in (Fig 3). It is shown that the amplitude of the vibrations did not make any noticeable change where the highest amplitude was found at the two support ends, taking into consideration that the frequency started decreasing slightly with the increase in crack width.

### 4.1.3 EFFECT OF SPEED VARIATION

The seventh mode of a cracked shaft supported by two bearings, at different speeds (50 rpm, 100 rpm, 250 rpm, 500 rpm, 750 rpm, 1500 rpm, 3000 rpm, and 6000 rpm) with the same crack position at the center of the shaft and the same crack dimensions at (1mm depth and 0.1 mm width) shown in (Fig 4). It is shown that the amplitude at the low speeds was low and almost constant but as soon as the speed started increasing the amplitude jumped to a higher value and stayed at that value as the speed was increased, as for the frequency of the shaft it was noticed that it decreased slightly with the increase in speed.

## 4.2 TWO BEARINGS WITH TWO CRACKS

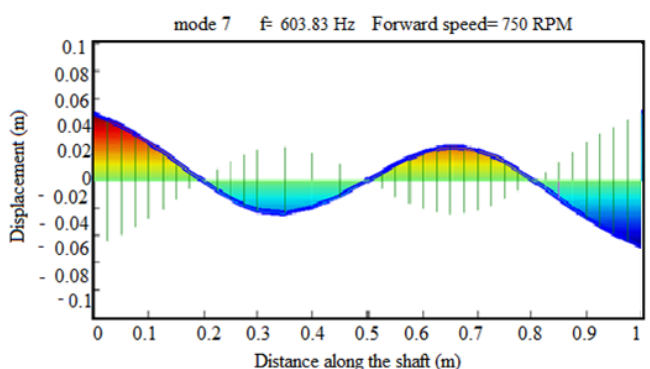


Fig (5) Displacement of shaft – Two Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.3m C<sub>2</sub>=0.7 N=750rpm)

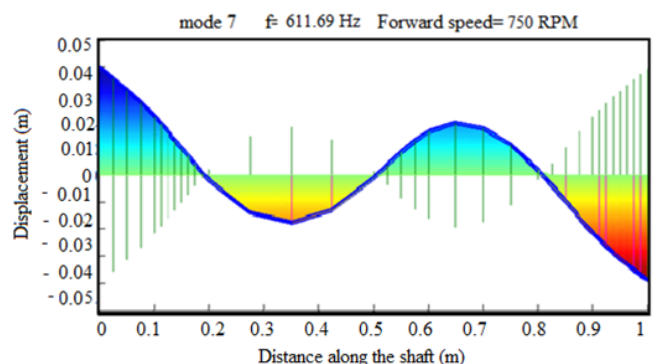


Fig (6) Displacement of shaft – Two Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.2m C<sub>2</sub>=0.8 N=750rpm)

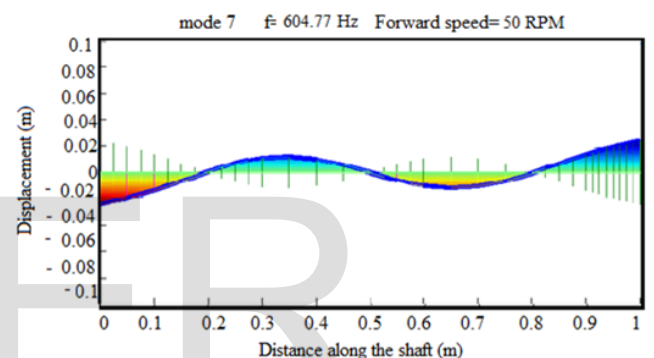


Fig (7) Displacement of shaft – Two Crack, Two bearings  
(a=1mm b=0.1mm C<sub>1</sub>=0.2m C<sub>2</sub>=0.8 N=50rpm)

### 4.2.1 EFFECT OF CRACK POSITION VARIATION

The seventh mode of a double cracked shaft supported by two bearings at different crack positions [(0.3m,0.7m), (0.4m,0.8m), (0.2m, 0.8m), and (0.1m, 0.9m)] consequently with fixed parameters of the rotational speed of the shaft at (750 rpm), and the crack dimensions constant at (1mm depth and 0.1mm width) shown in Fig(5). It is shown that the amplitude of the vibration changes according to the distance of the cracks from the supports as the amplitude decreases slightly with the cracks coming closer to the support ends, taking into consideration that the frequency increases as the cracks move away from the center.

### 4.2.2 EFFECT OF CRACK WIDTH VARIATION

The seventh mode of a double cracked shaft supported by two bearings, at different crack dimensions by changing the width of the crack (0.1mm, 0.2mm, 0.3mm, 0.4mm, 0.5mm, 0.7mm, and 1mm) at the same crack position at (0.2m, 0.8m) and crack depth at (1mm), and constant shaft rotation speed at (750 rpm)

shown in Fig(6). It is shown that the amplitude of the vibrations slightly increased with the increase in the crack width with the highest amplitude present at the support ends, taking into consideration that there were only slight changes in the frequency (almost constant).

#### 4.2.3 EFFECT OF SPEED VARIATION

The seventh mode of a double cracked shaft supported by two bearings, at different speeds (50 rpm, 100 rpm, 250 rpm, 500 rpm, 750 rpm, 1500 rpm, 3000 rpm, and 6000 rpm) with the same crack position at (0.2m, 0.8m) and the same crack dimensions at (1mm depth and 0.1 mm width) shown in Fig(7). It is shown that the amplitude at the low speeds was low but as soon as the speed started increasing the amplitude jumped to a higher value and stayed at that value as the speed was increased, as for the frequency of the shaft it was noticed that it decreased slightly with the increase in speed.

### 4.3 THREE BEARINGS WITH TWO CRACKS

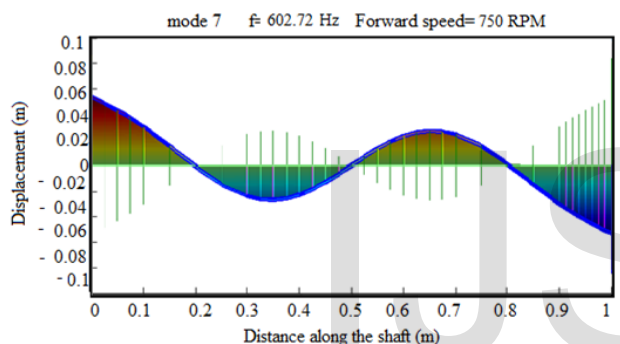


Fig (8) Displacement of shaft – Two Crack, Three bearings  
 (a=1mm b=0.1mm C<sub>1</sub>=0.3m C<sub>2</sub>=0.7 N=750rpm)

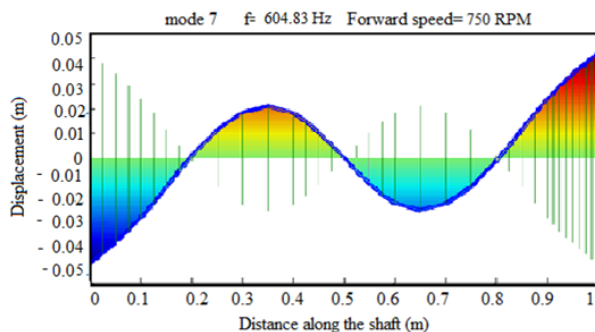


Fig (9) Displacement of shaft – Two Crack, Three bearings  
 (a=1mm b=0.1mm C<sub>1</sub>=0.2m C<sub>2</sub>=0.8 N=750rpm)

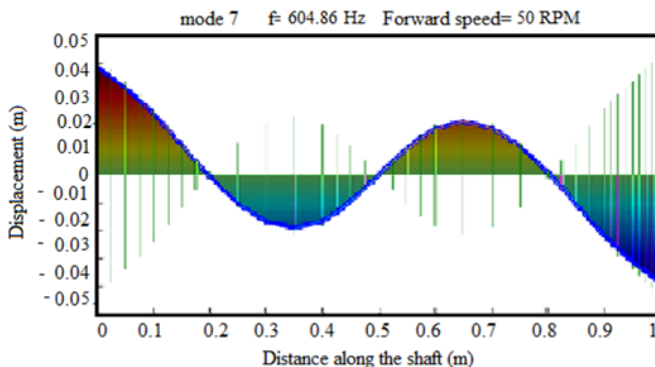


Fig (10) Displacement of shaft – Two Crack, Three bearings  
 (a=1mm b=0.1mm C<sub>1</sub>=0.2m C<sub>2</sub>=0.8 N=50rpm)

#### 4.3.1 EFFECT OF CRACK POSITION VARIATION

The seventh mode of a double cracked shaft supported by three bearings, at different crack positions [(0.3m,0.7m), (0.4m,0.8m), (0.2m, 0.8m), and (0.1m, 0.9m)] consequently with fixed parameters of the rotational speed of the shaft at (750 rpm), and the crack dimensions constant at (1mm depth and 0.1mm width) shown in Fig(8). It is shown that the amplitude of the vibration changes according to the distance of the cracks from the supports as the amplitude showed the highest value at the cracks located at the center between each two bearings with the cracks coming closer to the support ends the amplitude decreases, taking into consideration that the frequency increases as the cracks move away from the center.

#### 4.3.2 EFFECT OF CRACK WIDTH VARIATION

The seventh mode of a double cracked shaft supported by three bearings, at different crack dimensions by changing the width of the crack (0.1mm, 0.2mm, 0.3mm, 0.4mm, 0.5mm, 0.7mm, and 1mm) at the same crack position at (0.2m, 0.8m) and crack depth at (1mm), and constant shaft rotation speed at (750 rpm) shown in Fig(9). It is shown that the amplitude of the vibrations barely changed with the change in the crack width with the highest amplitude present at the support ends, taking into consideration that there were only slight decrease in the frequency as the crack width increased.

#### 4.3.3 EFFECT OF SPEED VARIATION

The seventh mode of a double cracked shaft supported by three bearings, at different speeds (50 rpm, 100 rpm, 250 rpm, 500 rpm, 750 rpm, 1500 rpm, 3000 rpm, and 6000 rpm) with the same crack position at (0.2m, 0.8m) and the same crack dimensions at (1mm depth and 0.1 mm width) shown in Fig(10). It is shown that the amplitude at the low speeds was slightly lower than that at high speeds, as for the frequency of the shaft it was noticed that it decreased slightly with the increase in speed.

#### 4.4 TWO BEARINGS WITH ONE CRACK AND A SINGLE MASS

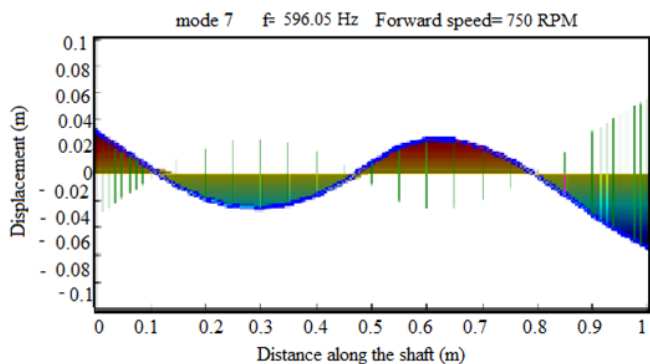


Fig (11) Displacement of shaft –One crack, Two Bearings, Single mass  
(a=1mm b=0.1 C1=0.5 C2=0 N=750rpm m=5kg p=0.1m)

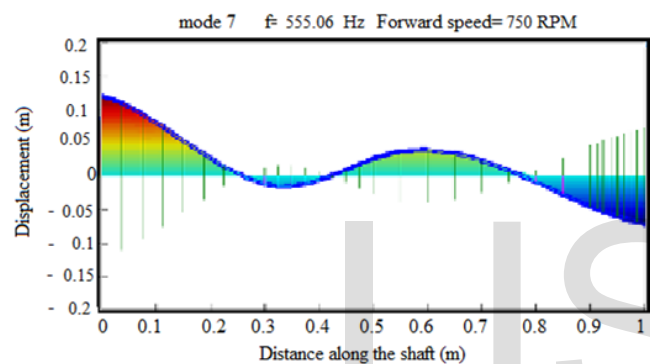


Fig (12) Displacement of shaft –One crack, Two Bearings, Single mass  
(a=1mm b=0.1 C1=0.5 C2=0 N=750rpm m=5kg p=0.3m)

##### 4.4.1 SINGLE MASS POSITION VARIATION

The seventh mode of a cracked shaft supported by two bearings, with a point load on the shaft changing its position along the shaft (0.1m, 0.3m, 0.5m, 0.7m, 0.9m) with fixed parameters of the rotational speed of the shaft at (750 rpm), and the crack dimensions constant at (1mm depth and 0.1mm width) at the center of the shaft shown in Fig(11). It is shown that the amplitude of the vibrations changes with the change in the position of the load, where the amplitude at the location of the load is low and the highest values of the amplitude are at the bearings sides.

##### 4.4.2 SINGLE MASS LOAD VARIATION

The seventh mode of a cracked shaft supported by two bearings, with a point load on the shaft at (0.3m) changing its value (5kg, 7kg, 10kg, 20kg, 30kg) with fixed parameters of the rotational speed of the shaft at (750 rpm), and the crack dimensions constant at (1mm depth and 0.1mm width) at the center of the shaft shown in Fig(12). It is shown that the amplitude of the vibrations changes with the change in the value of the load, where the amplitude at the load is very low, and the highest amplitude is at the bearing end closer to the load.

#### 4.5 COMBINED GRAPHICAL REPRESENTATION OF RESULTANTS

##### FOR ALL CASES

Fig (13) shows the effect of crack position on the maximum amplitude of the shaft when the speed of the shaft is constant and the width of the crack is the same at each position. For a specific point on the shaft it is shown that the maximum amplitude is highest when there was only 1 crack involved while the presence of two cracks shows a decrease in the maximum amplitude, there is a quiet good resemblance between the case with two cracks and three bearings and the case of two cracks with two bearings

Fig (14) Illustrates the relation between the maximum amplitude along the shaft when the crack is created in the shaft with different crack widths (0.1-1mm) Variations in the maximum amplitude between the cases show that the amplitude decreases with the increase in the number of cracks, taking into consideration that the results for having two cracks on a shaft supported by two bearings or three bearings doesn't differ a lot.

Fig (15) shows the relation between the maximum amplitude of the shaft and its speed is shown in the above figure, where it is shown that the maximum amplitude in the 2 cases of no crack and the presence of one crack can only be identified at low speeds, at high speeds the results are almost the same, while the cases with two cracks are close to each other in results they are noticeably lower in the maximum amplitude than the case with no crack.

Fig (16) shows the relation between the natural frequency and the change in the crack position. Where the natural frequency of the un-cracked shaft is constant along the shaft, while that of the cracked shafts have different values along the shaft, in addition to that the natural frequencies of the cracked shafts is in most cases lower than the frequency in the case of the un-cracked shaft.

Fig (17) shows the relation between the natural frequency of the rotating shaft and the change in the width of the crack. The natural frequency for the case of the no crack and 2 bearings with one crack are almost identical and very hard to differentiate while in the other two cases of the two cracks the results are also close but they can be differentiated from the un-cracked case, it can be noticed that the frequency decreases with the increase in the number of cracks.

Fig (18) represents the relation between the natural frequency of the rotating shaft and the shaft rotational speed, where it is shown that also for the case of the un-cracked shaft and the shaft with only one crack the results are very close, while the results for the shafts with 2 cracks are of lower frequency than the un-cracked case by a noticeable value, but there isn't that much difference between the case of the shaft with two bearings and two cracks and the case of the shaft with three bearings and two cracks.

Fig (19) shows that the amplitude changes with the change in the load position at constant crack parameters, where the highest values occur when the load is at the center distance between the crack and the supports. The amplitude increases

as the load moves away from the support till it reaches it maximum value at the center distance between the crack and the support, then starts to decrease till it reaches the crack and the same for the other half of the shaft.

Fig (20) shows that the amplitude changes with the change in the load value as the load value increases the amplitude increases. At small masses the increase in amplitude is noticeable while at relatively large masses the increase in amplitude becomes very small.

Fig (21) shows that the maximum natural frequency occurs when the crack is located at the center of the shaft, as the load moves away from the supports the amplitudes starts decreasing till it reaches the middle distance between the crack and the support its starts to increase till reaching the peak at the crack.

Fig (22) shows that the frequency decreases with the increase in the load acting on the shaft, where it decreased at a high range at the beginning then started to decrease gradually. It was noticed that as the frequency decreases with the increase in the load the maximum amplitude increases with the increase in the load

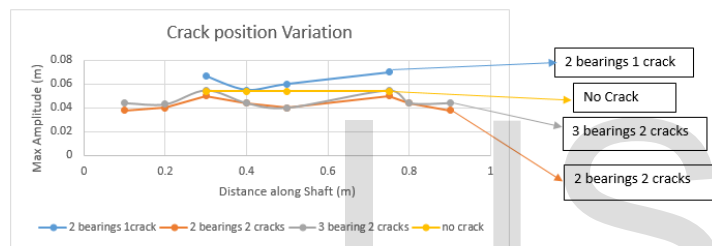


Fig (13) Effect of crack position on maximum amplitude of vibrations (two bearings-Three bearings)

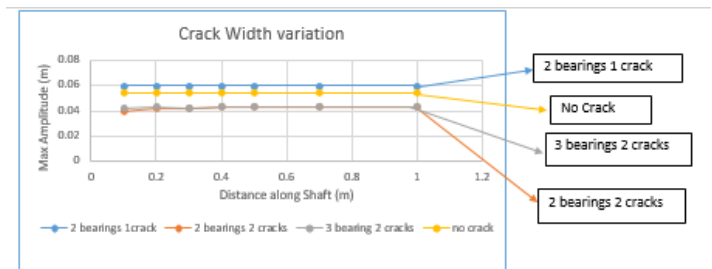


Fig (14) Maximum amplitude of the shaft at variable crack width (0.1-1) mm

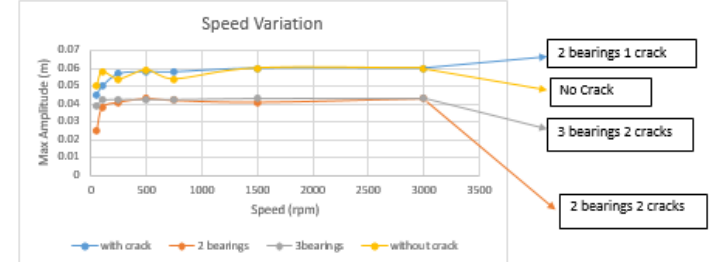


Fig (15) Effect of shaft speed on maximum amplitude at constant crack properties

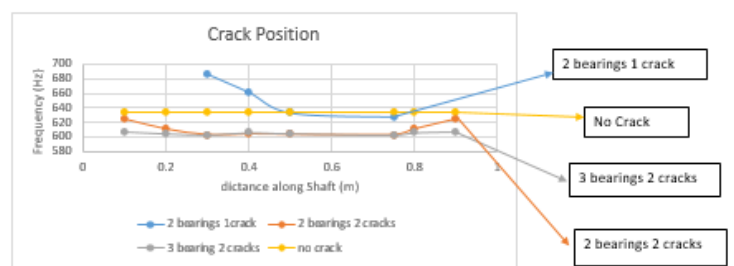


Fig (16) Effect of crack position on the natural Frequency

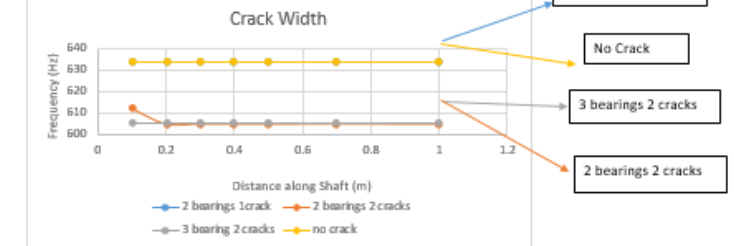


Fig (17) the effect of crack width on the natural Frequency

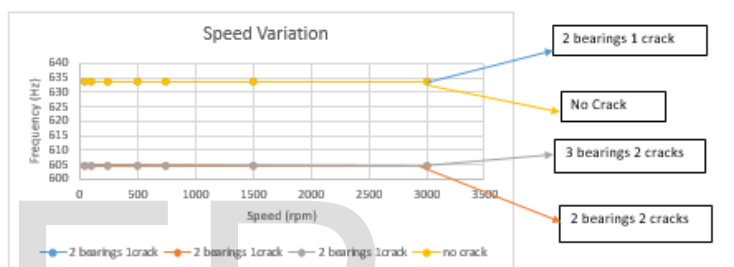


Fig (18) Effect of Shaft speed on the natural Frequency

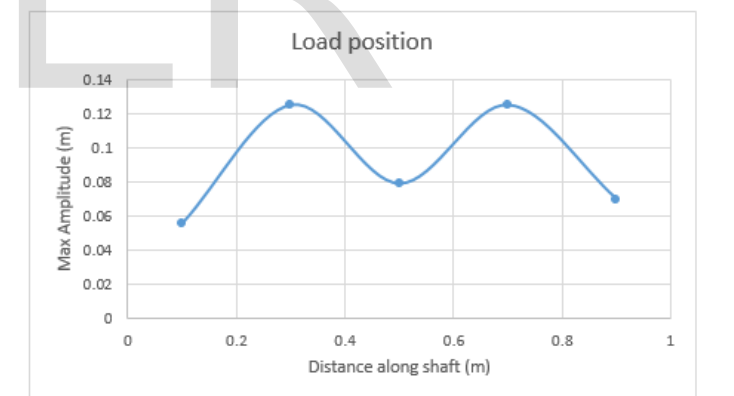


Fig (19) Effect of load position on the maximum amplitude

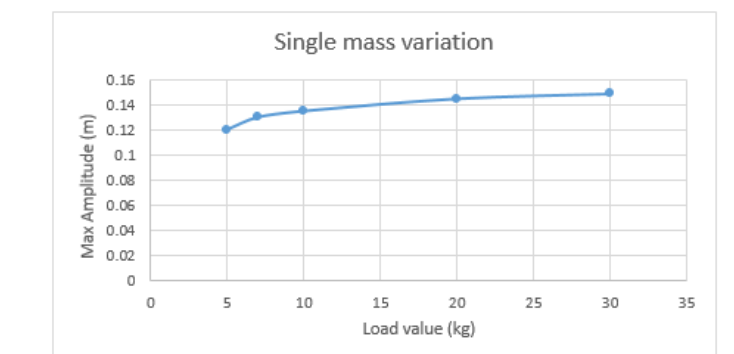


Fig (20) Effect of load value on maximum amplitude at constant crack properties



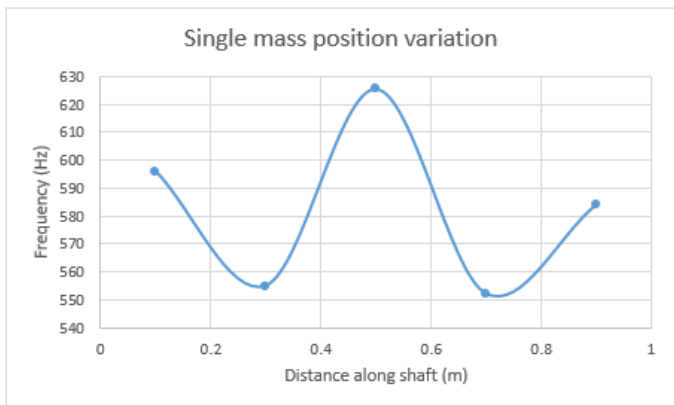


Fig (21) Effect of load position on the natural frequency of a cracked shaft

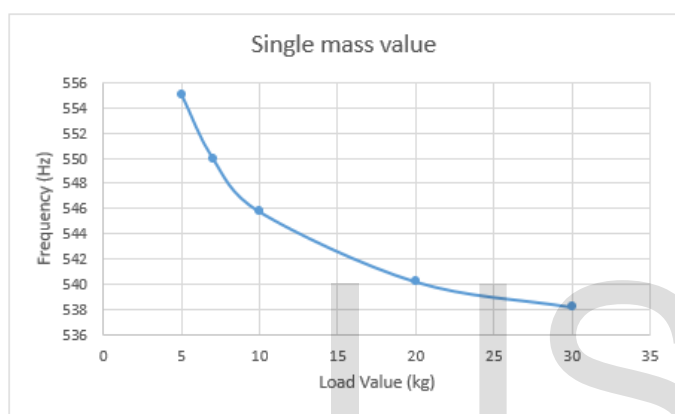
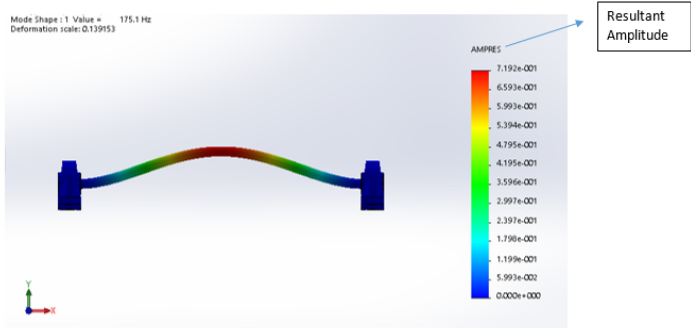


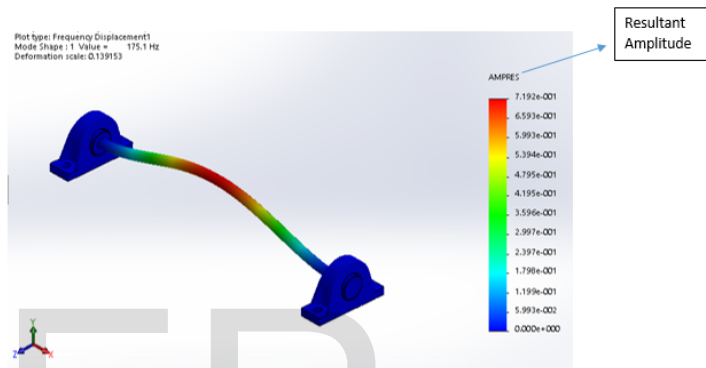
Fig (22) Effect of load value on the natural frequency

#### 4.6 SOLIDWORKS SIMULATION RESULTS

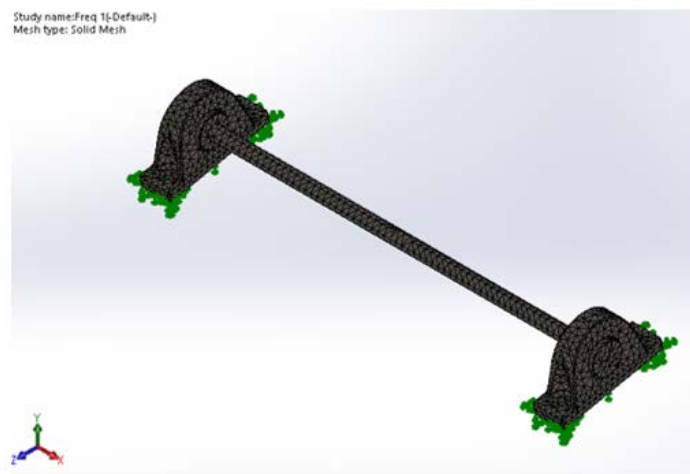
Shows the results from simulating a crack in a rotating shaft on SolidWorks, where each mode illustrates the deformation shown due to fatigue at each mode's natural frequency. It is shown that the resultant amplitude varies from 0.071 at the first mode to 0.068. When comparing the SolidWorks results to the results obtained from the Matlab simulation it can be noticed that the Matlab results at the same crack and shaft properties are almost 0.062 while that of the SolidWorks is 0.068



Crack depth 1mm crack width 0.1mm at the center (750 rpm) Mode 1 Isometric View



Crack depth 1mm crack width 0.1mm at the center (750 rpm) Frequency displacement for Mode 1 Isometric View



Crack depth 1mm crack width 0.1mm at the center Solid mesh view

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